GCE A LEVEL MARKING SCHEME

SUMMER 2024

A LEVEL
MATHEMATICS
UNIT 3 PURE MATHEMATICS B
1300U30-1

About this marking scheme

The purpose of this marking scheme is to provide teachers, learners, and other interested parties, with an understanding of the assessment criteria used to assess this specific assessment.

This marking scheme reflects the criteria by which this assessment was marked in a live series and was finalised following detailed discussion at an examiners' conference. A team of qualified examiners were trained specifically in the application of this marking scheme. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners. It may not be possible, or appropriate, to capture every variation that a candidate may present in their responses within this marking scheme. However, during the training conference, examiners were guided in using their professional judgement to credit alternative valid responses as instructed by the document, and through reviewing exemplar responses.

Without the benefit of participation in the examiners' conference, teachers, learners and other users, may have different views on certain matters of detail or interpretation. Therefore, it is strongly recommended that this marking scheme is used alongside other guidance, such as published exemplar materials or Guidance for Teaching. This marking scheme is final and will not be changed, unless in the event that a clear error is identified, as it reflects the criteria used to assess candidate responses during the live series.

WJEC GCE A LEVEL MATHEMATICS

UNIT 3 PURE MATHEMATICS B

SUMMER 2024 MARK SCHEME

Q	Solution	Mark	Notes
1(a)	$\frac{25x+32}{(2x-5)(x+1)(x+2)} = \frac{A}{(2x-5)} + \frac{B}{(x+1)} + \frac{C}{(x+2)}$	M1	
	$25x + 32 = A(x+1)(x+2) + B(2x-5)(x+2) + C(2x-5)(x+1)$	m1	correct removal of denom, oe, si
	Put $x = -1$, $-25 + 32 = B(-7)(1)$		
	$B = -1$	A1	one constant correct
	Put $x = -2$, $-50 + 32 = C(-9)(-1)$		
	$C = -2$		
	Coefficient x^2 (or put $x = 2.5$),		oe
	$0 = A + 2B + 2C = A - 2 - 4$		
	$A = 6$	A1	all three constants correct
	$f(x) = \frac{6}{(2x-5)} - \frac{1}{(x+1)} - \frac{2}{(x+2)}$		
1(b)	$\int_1^2 \left(\frac{6}{(2x-5)} - \frac{1}{(x+1)} - \frac{2}{(x+2)}\right) \mathrm{d}x$	M1	FT at least 2 of their non-zero *A*, *B*, *C*, limits not required, si
	$= \left[\frac{6}{2}\ln\|2x-5\| - \ln\|x+1\| - 2\ln\|x+2\|\right]_1^2$	A1	for one correct term, FT *A*, *B*, *C*.
		A1	all 3 correct, FT *A*, *B*, *C* condone brackets instead of ||.
	$= [3\ln\|-1\| - \ln 3 - 2\ln 4] - [3\ln\|-3\| - \ln 2 - 2\ln 3]$	m1	correct use of limits, si
	$= -2\ln 3 - 3\ln 2$		
	$= -\ln 72$	A1	cao, $P = 72$

Q	Solution	Mark	Notes
1(c)	$f(2) = -\frac{82}{12} < 0$		
	$f(3) = \frac{107}{20} > 0$	B1	
	Denominator = 0 when $x = 2.5$.	E1	oe Eg (vertical) asymptote at $x = 2.5$. Function discontinuous when $x = 2.5$.

Q	Solution	Mark	Notes
2(a)	$3\cot\theta + 4(1 + \cot^2\theta) = 5$	M1	Use of $\text{cosec}^2\theta = 1 + \cot^2\theta$
	$4\cot^2\theta + 3\cot\theta - 1 = 0$		
	$(4\cot\theta - 1)(\cot\theta + 1) = 0$	m1	coeff $\cot\theta$ multiply to 4, constant terms multiply to -1
	$\cot\theta = -1, \frac{1}{4}$	A1	cao, both values
	$\tan\theta = -1$		
	$x = 135°, 315°$	B1	ft only if $\tan\theta$ is correct from $\cot\theta$. or allow one correct value from each branch $2.326^c, 5.498^c$
	$\tan\theta = 4,$		
	$x = 75.96°, 255.96°$	B1	ft if sign different $1.326^c, 4.467^c$

<u>Note</u>

Ignore all roots outside range.

For each branch, award B0 if extra root(s) in range present.

2 +ve roots ft for B1, 2 -ve roots ft for B1.

Only award for $\tan\theta$.

If radians used, units are required.

Q	Solution	Mark	Notes
2(a)	OR		
	$3\cot\theta + 4(1 + \cot^2\theta) = 5$	(M1)	Use of $\text{cosec}^2\theta = 1 + \cot^2\theta$
	$4\cot^2\theta + 3\cot\theta - 1 = 0$		
	multiplying by $\tan^2\theta$ $4 + 3\tan\theta - \tan^2\theta = 0$, $\tan^2\theta - 3\tan\theta - 4 = 0$ $(\tan\theta - 4)(\tan\theta + 1) = 0$	(m1)	coeff $\tan\theta$ multiply to 1, constant terms multiply to –4
	$\tan\theta = -1, 4$	(A1)	cao
	$\tan\theta = -1$,		
	$\theta = 135°, 315°$	(B1)	ft or allow one correct value from each branch $2.326^c, 5.498^c$
	$\tan\theta = 4$,		
	$\theta = 75.96°, 255.96°$	(B1)	ft if sign different, $1.326^c, 4.467^c$

<u>Note</u>

Ignore all roots outside range.

For each branch, award B0 if extra root(s) in range present.

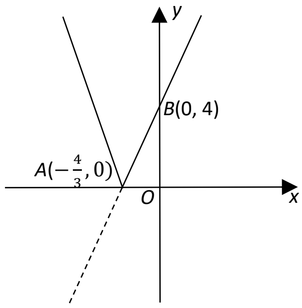
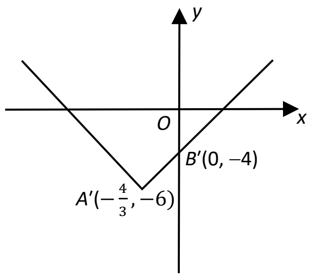
2 +ve roots ft for B1, 2 -ve roots ft for B1.

Only award for $\tan\theta$.

If radians used, units are required.

Q	Solution	Mark	Notes
2(b)	$24\cos x - 7\sin x = R\cos(x + \alpha)$		
	$= R\cos x\cos\alpha - R\sin x\sin\alpha$		
	$R\cos\alpha = 24$, $R\sin\alpha = 7$	M1	implied by $\alpha = 16.26°$
	$R = \sqrt{24^2 + 7^2} = 25$	B1	no working required
	$\alpha = \tan^{-1}\left(\frac{7}{24}\right)$		$\sin^{-1}\left(\frac{7}{25}\right)$, $\cos^{-1}\left(\frac{24}{25}\right)$
	$\alpha = 16.26°$	A1	cao
	$25\cos(x + 16.26°) = 16$	M1	ft R, α
	$\cos(x + 16.26°) = \frac{16}{25} = 0.64$		
	$x + 16.26° = 50.21°, 309.79°$	A1	any one value, ft R, α, si
	$x = 33.95°, 293.53°$	A1	ft R, α, both values. A0 if 3 or more values between $0° < x < 360°$. Accept whole numbers. Ignore values outside the range.

Q	Solution	Mark	Notes
3(a)	Area of sector = $\frac{1}{2}r^2\theta$	B1	used si
	$\frac{1}{2}(5+r)^2 \times \frac{\pi}{5} - \frac{1}{2}r^2 \times \frac{\pi}{5} = \frac{13\pi}{2}$	M1	difference between two sectors Condone wrong way round.
		A1	correct equation
	$r^2 + 10r + 25 - r^2 = 65$		
	$10r + 25 = 65$		
	$2r + 5 = 13$		
	$r = 4$	A1	cao
			FT their part (a)
3(b)	length $CD = \frac{(5+r)\pi}{5} = \frac{9\pi}{5}$		
	length $AB = \frac{r\pi}{5} = \frac{4\pi}{5}$	B1	si, either correct with r substituted
	Perimeter = 5 + 5 + CD + AB	M1	
	Perimeter = $5 + 5 + \frac{4\pi}{5} + \frac{9\pi}{5}$		
	Perimeter = $10 + \frac{13\pi}{5}$ (= 18.168…)	A1	cao, isw

Q	Solution	Mark	Notes
4(a)		G1	shape, above the *x*-axis and vertex on *x*-axis
		B1	$\left(-\frac{4}{3}, 0\right)$, any correct method
		B1	(0, 4), any correct method
4(b)		G1	move downwards, ft (a)
		B1	$\left(-\frac{4}{3}, -6\right)$, ft their $\left(-\frac{4}{3}, 0\right)$, any correct method
		B1	(0, −4) , any correct method

Q	Solution	Mark	Notes
5	(Assume that there is a real and positive value of x such that)		
	$x + \frac{81}{x} < 18.$	M1	condone $\leq$
	Then		
	$x^2 + 81 < 18x$	A1	
	$x^2 - 18x + 81 < 0$		
	$(x - 9)^2 < 0$	A1	
	which is impossible, hence contradiction		
	Therefore $x + \frac{81}{x} \geq 18.$	A1	cso

Q	Solution	Mark	Notes
6(a)	$y = \cos x$		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \lim_{h\to 0} \frac{\cos(x+h) - \cos x}{h}$	M1	
	$= \lim_{h\to 0} \frac{\cos x \cos h - \sin x \sin h - \cos x}{h}$	B1	$\frac{\cos x \cos h - \sin x \sin h - \cos x}{h}$
	As $h\to 0$, $\sin h \cong h$ and $\cos h \cong 1 - \frac{h^2}{2}$		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \lim_{h\to 0} \frac{\cos x\left(1 - \frac{h^2}{2}\right) - h\sin x - \cos x}{h}$	M1	substitution
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \lim_{h\to 0} \frac{\cos x\left(-\frac{h^2}{2}\right) - h\sin x}{h}$	A1	
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \lim_{h\to 0}\left(-\frac{h}{2}\cos x - \sin x\right)$		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = -\sin x$	A1	Everything correct & convincing

Q	Solution	Mark	Notes
	OR		
6(a)	$y = \cos x$		
	$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\cos(x + \delta x) - \cos x}{\delta x}$	(M1)	
	$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\cos x \cos \delta x - \sin x \sin \delta x - \cos x}{\delta x}$	(B1)	$\frac{\cos x \cos \delta x - \sin x \sin \delta x - \cos x}{\delta x}$
	$\frac{dy}{dx} = \lim_{\delta x \to 0} \left[\frac{\cos x(\cos \delta x - 1)}{\delta x} - \frac{\sin x \sin \delta x}{\delta x}\right]$	(M1)	factorisation
	$\frac{dy}{dx} = \cos x \lim_{\delta x \to 0} \left[\frac{\cos \delta x - 1}{\delta x}\right]$ $- \sin x \lim_{\delta x \to 0} \left[\frac{\sin \delta x}{\delta x}\right]$		
	But $\lim_{\delta x \to 0} \left[\frac{\cos \delta x - 1}{\delta x}\right] = 0$		
	and $\lim_{\delta x \to 0} \left[\frac{\sin \delta x}{\delta x}\right] = 1$	(A1)	both limits used
	Hence $\frac{dy}{dx} = \cos x \times 0 - \sin x \times 1$		
	$\frac{dy}{dx} = -\sin x$	(A1)	Everything correct & convincing

Q	Solution	Mark	Notes
6(b)	$y = e^{3x}\sin 4x$		
	$\frac{dy}{dx} = e^{3x} \times 4\cos 4x + 3e^{3x} \times \sin 4x$	M1	$e^{3x}f(x) + g(x)\sin 4x$, $f(x), g(x) \neq 0, 1$.
		A1	$f(x) = 4\cos 4x$, isw
		A1	$g(x) = 3e^{3x}$, isw
	$\frac{dy}{dx} = e^{3x}(4\cos 4x + 3\sin 4x)$		

Q	Solution	Mark	Notes
6(c)	$\int x^2 \sin 2x \, dx$		
	$= \left[-\frac{1}{2}\cos 2x \times x^2\right] - \int -\frac{1}{2}\cos 2x \times 2x \, dx$	M1	$[f(x)x^2] - \int f(x)g(x)dx$, $g(x) \neq 0,1$, condone a sign error. $f(x) \neq \sin 2x$
		A1	$f(x) = -\frac{1}{2}\cos 2x$
		A1	$g(x) = 2x$
	$= \left[-\frac{1}{2}x^2\cos 2x\right] + \left[\frac{1}{2}\sin 2x \times x\right] - \int \frac{1}{2}\sin 2x \times 1 dx$		
		A1	correct second and third terms
	$= \left[-\frac{1}{2}x^2\cos 2x\right] + \left[\frac{1}{2}x\sin 2x\right]$		
	$+ \left[\frac{1}{4}\cos 2x\right] + \text{C}$	A1	third term correct and + C. isw

Q	Solution	Mark	Notes
7(a)	$\sum_{r=3}^{50}(4r+5)$		
	= 17 + 21 + 25 + … (+ 205)	M1	oe AP recognised
	$=\frac{48}{2}[2\times 17+(48-1)\times 4]$	m1	use of formula, condone 1 error.
			$\frac{48}{2}[17+205]$
		A1	all correct
		Accept a = 13, d = 4, n = 49, Sum = 5341	
		Accept a = 9, d = 4, n = 50, Sum = 5350	
		Accept a = 5, d = 4, n = 51, Sum = 5355	
	= 5328	A1	cao, from correct working
	Note: No workings, M0		

Q	Solution	Mark	Notes
7(b)	$\sum_{r=2}^{\infty}\left(540\times\left(\frac{1}{3}\right)^r\right)$		
	$= 540\left(\frac{1}{3^2}+\frac{1}{3^3}+\frac{1}{3^4}+\cdots\right)$	M1	GP recognised
	$= 60 + 20 + \frac{20}{3} + \ldots$		
	$=\frac{540\times\frac{1}{9}}{1-\frac{1}{3}}=\frac{60}{1-\frac{1}{3}}$	m1	correct use of formula, ratio = $\frac{1}{3}$
			Accept $\frac{540\times\frac{1}{3}}{1-\frac{1}{3}}=\frac{180}{1-\frac{1}{3}}$, Sum = 270
			Accept $\frac{540}{1-\frac{1}{3}}$, Sum = 810
	$= 90$	A1	cao

<u>Note</u>: No workings, M0

Q	Solution	Mark	Notes
8(a)	$f(0) = -1 < 0$		
	$f(1) = 1 > 0$		
	$f(x)$ changes sign in the interval [0, 1],		
	(and f is continuous / f is a cubic curve.)	B1	
	Hence, there is a root in the interval [0, 1]		
8(b)(i)	$f'(x) = 3x^2 + 8x - 3$	B1	si
	$x_{n+1} = x_n - \frac{x_n^3 + 4x_n^2 - 3x_n - 1}{3x_n^2 + 8x_n - 3}$	M1	$x_{n+1} = \frac{2x_n^3 + 4x_n^2 + 1}{3x_n^2 + 8x_n - 3}$, si
	$x_0 = 0.8$		
	$x_1 = 0.861\ 654\ 135\ 3\ldots$	A1	cao
8(b)(ii)	$x_2 = 0.857\ 641\ 074\ 1\ldots$		
	$x_3 = 0.857\ 623\ 607\ 4\ldots$		
	$x_4 = 0.857\ 623\ 607\ldots$		
	Root = 0.857 624 (correct to 6 dp)	A1	cao

Q	Solution	Mark	Notes
8(c)	$f'(x) = 3x^2 + 8x - 3$		
	$f'\left(\frac{1}{3}\right) = 3 \times \frac{1}{9} + 8 \times \frac{1}{3} - 3 = 0$	B1	$f'(x) = 3x^2 + 8x - 3 = (3x - 1)(x + 3)$
			When $x = \frac{1}{3}$, $f'(x) = 0$
	Therefore N-R requires division by 0.	E1	oe eg tangent at $x = \frac{1}{3}$ is horizontal, hence not intersecting the x-axis for the next iteration.

Q	Solution	Mark	Notes
9	Area under $C_1 = \int_0^{\pi}(-x^2 + \pi x + 1)\mathrm{d}x$	M1	Attempt to integrate, index increased in at least one term.
	$= \left[-\frac{x^3}{3} + \frac{\pi x^2}{2} + x\right]_0^{\pi}$	B1	correct integration, limits not required
	$= -\frac{\pi^3}{3} + \frac{\pi^3}{2} + \pi$	m1	correct use of limits, si
	$= \pi + \frac{\pi^3}{6}$	A1	cao, si, allow $-\frac{\pi^3}{3} + \frac{\pi^3}{2} + \pi$
	Area under C_2		
	$= \int_0^{\frac{\pi}{4}}(\cos 2x)\mathrm{d}x + \int_{\frac{3\pi}{4}}^{\pi}(\cos 2x)\mathrm{d}x$	M1	oe, either integral
	$= 2\left[\frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{4}}$	B1	correct integration of $\cos 2x$, limits not required
	$= 2\left(\frac{1}{2} - 0\right)$	(m1)	si correct use of limits
	$= 1$	A1	cao, si or each integral = $\frac{1}{2}$, both required.

Note: Award the m1 once for correct substitution of limits seen in either integration.

	Required area		
	= Area under C_1 – Both Areas under C_2	m1	used
	Required area = $\pi + \frac{\pi^3}{6} - 1$	A1	cao, exact value required

Note: SC3 for answer only of 7.3093.

SC2 for answer only of 8.3093 AND ½ or 1.

SC1 for answer only of 8.3093 OR ½ or 1

Q	Solution	Mark	Notes
9	Alternative solution		
	Area between curves		
	$= \int_0^{\pi}((-x^2 + \pi x + 1) - \cos 2x)\mathrm{d}x$	M1	
	$= \left[-\frac{x^3}{3} + \frac{\pi x^2}{2} + x - \frac{1}{2}\sin 2x\right]_0^{\pi}$	B1	correct integration of quadratic
		B1	correct integration of $\cos 2x$
	$= \left(-\frac{\pi^3}{3} + \frac{\pi^3}{2} + \pi - \frac{1}{2}\sin 2\pi\right) - 0$	m1	correct use of limits, si
	$= \pi + \frac{\pi^3}{6}$	A1	cao, si
	Area below x-axis		
	$= \left\lvert \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} (\cos 2x)\mathrm{d}x \right\rvert$	M1	
	$= \left\lvert \left[\frac{1}{2}\sin 2x\right]_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \right\rvert$	(B1)	correct integration of $\cos 2x$
	$= \left\lvert \left[\frac{1}{2}\sin\frac{3\pi}{2}\right] - \left[\frac{1}{2}\sin\frac{\pi}{2}\right] \right\rvert$	(m1)	si
	$= \left\lvert -\frac{1}{2} - \frac{1}{2} \right\rvert$		
	$= 1$	A1	cao, si, condone -1 if used correctly below

Note: Award the m1 once for correct substitution of limits seen in either integration.

	Required area		
	= Area between curves − Area below x-axis	m1	
	Required area $= \pi + \frac{\pi^3}{6} - 1$	A1	cao, exact value required

Note: SC3 for answer only of 7.3093.

SC2 for answer only of 8.3093 AND (−)1.

SC1 for answer only of 8.3093 OR (−)1.

Q	Solution	Mark	Notes
10(a)	$\frac{2(3x+1)}{x^2-2x-3}+\frac{x}{x+1}=\frac{2(3x+1)}{(x-3)(x+1)}+\frac{x}{x+1}$	B1	factorise denominator
	$=\frac{2(3x+1)+x(x-3)}{(x-3)(x+1)}$	B1	common denominator
	$=\frac{x^2+3x+2}{(x-3)(x+1)}$	B1	simplify numerator
	$=\frac{(x+1)(x+2)}{(x-3)(x+1)}=\frac{x+2}{x-3}$	B1	factorise and cancel ($x \neq -1$). AG
10(b)	$f(x)=\frac{x+2}{x-3}=1+\frac{5}{x-3}$		
	Range is (1, 6]	B1	(1, oe
		B1	6] oe
			SC1 [6, 1)
10(c)	$y=\frac{x+2}{x-3}$	M1	x and y may be interchanged at start
	$xy-3y=x+2$		
	$xy-x=3y+2$	m1	attempt to isolate x (or y)
	$x(y-1)=3y+2$		
	$x=\frac{3y+2}{y-1}$		
	$f^{-1}(x)=\frac{3x+2}{x-1}$	A1	oe
	Domain (1, 6], range [4, ∞)	B1	FT part (b) only for domain, oe
10(d)	$\frac{x+2}{x-3}=\frac{3x+2}{x-1}$	M1	$x=\frac{3x+2}{x-1}$; $x=\frac{x+2}{x-3}$; $\frac{\frac{x+2}{x-3}+2}{\frac{x+2}{x-3}-3}=x$ ft (c)
	$2x^2-8x-4=0$	A1	$x^2-4x-2=0$, ft (c) only if similar expression for $f^{-1}(x)$
	$x=2\pm\sqrt{6}$	A1	cao
	Since $x \geq 4$, $x=2+\sqrt{6}=4.449\ldots$	A1	cao
	Note: Answer only M1 A0 m0 A0		

Q	Solution	Mark	Notes
11(a)	$\dfrac{dy}{dx} = \dfrac{\dfrac{dy}{d\theta}}{\dfrac{dx}{d\theta}}$	M1	used, si
	$\dfrac{dx}{d\theta} = 2 + 2\cos 2\theta$	B1	
	$\dfrac{dy}{d\theta} = -2\sin 2\theta$	B1	
	$\dfrac{dy}{dx} = \dfrac{-2\sin 2\theta}{2(1+\cos 2\theta)}$		
	$\dfrac{dy}{dx} = \dfrac{-2\sin\theta\cos\theta}{1+2\cos^2\theta - 1}$	m1	$\sin 2\theta = 2\sin\theta\cos\theta$
		m1	$\cos 2\theta = 2\cos^2\theta - 1$
	$\dfrac{dy}{dx} = -\dfrac{\sin\theta}{\cos\theta} = -\tan\theta$	A1	convincing, AG
11(b)	Where $\theta = \frac{\pi}{4}$, point is $P\left(\left(\frac{\pi}{2}+1\right), 1\right)$	B1	si, $P(2.57, 1)$
	Gradient of tangent = $-\tan\frac{\pi}{4} = -1$	B1	si
	Equation of tangent is		
	$y - (1 + \cos 2\theta) = -\tan\theta(x - (2\theta + \sin 2\theta))$	M1	oe, method for equation, si e.g. $y = -1x + c$
	$y - 1 = -1\left(x - \left(\frac{\pi}{2}+1\right)\right)$	A1	oe, all correct, isw
	$y + x = 2 + \frac{\pi}{2}$ (= 3.57)		

Q	Solution	Mark	Notes
12(a)	For small θ, $\sin\theta \approx \theta$; $\cos\theta \approx 1 - \frac{\theta^2}{2}$.	M1	used
	$2\cos\theta + \sin\theta - 1$		
	$= 2\left(1 - \frac{\theta^2}{2}\right) + \theta - 1$		
	$= 2 - \theta^2 + \theta - 1$		
	$= 1 + \theta - \theta^2$	A1	AG
12(b)	$[1 + \theta - \theta^2]^{-1}$	M1	
	$= [1 + (\theta - \theta^2)]^{-1}$		
	$= 1 + (-1)(\theta - \theta^2) + \frac{(-1)(-2)}{2}(\theta - \theta^2)^2 + \cdots$	A1	$1 + (-1)(\theta - \theta^2)$
		A1	$\frac{(-1)(-2)}{2}(\theta - \theta^2)^2$
	$= 1 - \theta + \theta^2 + \theta^2 + \cdots$		
	$= 1 - \theta + 2\theta^2 + \cdots$	A1	cao
	$(a = -1, b = 2)$		

Q	Solution	Mark	Notes
13(a)	$f(x)$ is decreasing, so $f'(x) < 0$		
	$f(x)$ is convex, so $f''(x) > 0$,		
	$f'(x)$ is increasing.	E1	oe
	Required interval is $(x_3, 0)$	B1	condone $[x_3, 0]$, oe
13(b)	(Points of inflection occur when there is		
	a change of concavity,		
	ie change of sign in $f''(x)$.)		
	x-coordinates are x_3 or x_4	B1	either

Q	Solution	Mark	Notes
14(a)	$y = \frac{1+\ln x}{x}$		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{xf(x) - (1+\ln x)g(x)}{x^2}$	M1	Allow omission of $g(x)$
	$= \frac{x\left(\frac{1}{x}\right) - (1+\ln x)1}{x^2}$		$f(x) = \frac{1}{x}$, $(g(x) = 1)$
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-\ln x}{x^2}$	A1	convincing, AG
	<u>Alternative solution 1</u>		
	$y = \frac{1}{x} + \frac{\ln x}{x}$		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{x^2} - \frac{1}{x^2}\ln x + \frac{1}{x} \times \frac{1}{x}$	(M1)	one correct term
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-\ln x}{x^2}$	(A1)	convincing, AG
	<u>Alternative solution 2</u>		
	$y = (1 + \ln x)x^{-1}$		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = x^{-1}f(x) + (1 + \ln x)g(x)$	(M1)	
	$= x^{-1}\frac{1}{x} + (1 + \ln x) \times -1x^{-2}$		$f(x) = \frac{1}{x}$, $g(x) = -1x^{-2}$
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-\ln x}{x^2}$	(A1)	convincing, AG
	<u>Alternative solution 3</u>		
	$xy = 1 + \ln x$		
	$y + x\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{x}$	(M1)	
	$x\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{x} - y = \frac{1}{x} - \frac{1+\ln x}{x} = -\frac{\ln x}{x}$		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-\ln x}{x^2}$	(A1)	convincing, AG

Q	Solution	Mark	Notes
14(b)	$\int \frac{\ln x}{x^2}\,\mathrm{d}x = \int t\,\mathrm{d}t$	M1	variables separated, 1 error only.
	$-\frac{1+\ln x}{x} = \frac{t^2}{2}\ (+\ C)$	A1	$\frac{1+\ln x}{x}$
		A1	$\frac{t^2}{2}$
			−1 for incorrect signs if A1A1
	When $x = 1$, $t = 3$		
	$-1 = \frac{9}{2} + C$	m1	use of conditions
	$C = -\frac{11}{2}$		
	$t^2 = 11 - \frac{2(1+\ln x)}{x}$	A1	cao, oe, isw

Q	Solution	Mark	Notes
15	For £P invested, the return after n years is S.		
	Use of GP formula for kth term	M1	
	$S_A = P(1.01)^n$	B1	Allow $P = 1$
	$S_B = P(1.05)(1.006)^{n-1}$	B1	Allow $P = 1$
	We want minimum n st $S_A > S_B$		
	$P(1.01)^n > P(1.05)(1.006)^{n-1}$	m1	Allow $P = 1$. Condone ≥ or =
	$(1.006)(1.01)^n > (1.05)(1.006)^n$		
	$\left(\frac{1.01}{1.006}\right)^n > \left(\frac{1.05}{1.006}\right)$		
	$n\ln\left(\frac{1.01}{1.006}\right) > \ln\left(\frac{1.05}{1.006}\right)$		
	$n > 10.78(762515)$		
	min $n = 11$	A1	cao Mark final answer

Note: Accept any valid method (eg trial and error).

	Use of GP formula for kth term	M1	at least two iterations
	$S_A = P(1.01)^n$	B1	Allow $P = 1$, used for 2 values of n
	$S_B = P(1.05)(1.006)^{n-1}$	B1	Allow $P = 1$, used for 2 values of n
	Two iterations for n = 9, 10, 11, 12, 13	m1	
	min $n = 11$	A1	cao Mark final answer

	Bank *B*	**Bank *A***	
	$1.05 \times 1.006^{n-1}$	1.01^n	
$n = 9$	1.101 471 196…	1.093 685 272…	$B > A$
$n = 10$	1.108 080 023…	1.104 622 125…	$B > A$
$n = 11$	1.114 728 503…	1.115 668 346…	$B < A$
$n = 12$	1.121 416 874…	1.126 825 030…	$B < A$
$n = 13$	1.128 145 376…	1.138 093 280…	$B < A$

Therefore $n = 11$